

Polar and Geostationary Satellite Soundings – Experimental Product User’s Guide

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1. Introduction

Atmospheric radiance data from polar and geostationary orbiting satellites instruments can provide near-continuous high spatial and temporal resolution atmospheric temperature and humidity soundings on both global and regional scales. Direct Broadcast Satellite (DBS) Polar Hyperspectral Sounder (PHS), Cross-track Infrared Sounder (CrIS) and the Infrared Atmospheric Sounding Interferometer (IASI) atmospheric radiances are combined with polar satellite Microwave (MW) and Geostationary Satellite (GS) multispectral Advanced Baseline Imager (ABI) radiances to produce 2-km horizontal resolution temperature and humidity profiles, called ‘PHSnMWnABI’. Experimental forecasts results indicate that the high-spatial and temporal (30 to 60 min) resolution moisture measurements resolve the thermodynamic (i.e., atmospheric stability) and dynamic (i.e., horizontal, and vertical motions) processes responsible for localized severe weather. The satellite moisture profiles are continuously assimilated at hourly intervals into a 4-km High Resolution Rapid Refresh (RAP-like) Weather Research and Forecasting (WRF) model, to improve the skill of forecasting atmospheric state parameters, including 3-D winds, precipitation, and severe convective weather. The high-resolution satellite sounding/Numerical Weather Prediction (NWP) system has been operated in near real-time (24/7) for the past three years to experimentally demonstrate improvements in numerical forecasts of convective weather expected to result from using the satellite high-resolution sounding data in National Weather Service (NWS) operations. This Forecasters User’s Guide is intended to provide the basis for the high-resolution atmospheric profiles and the nowcasting and numerical forecasting products derived from them. Products are available through the NOAA AWIPS system, as well as being made available from University of Wisconsin, NASA/LaRC, and Hampton University websites.

2. Atmospheric Sounding Retrieval

The key elements of the ‘PHSnMWnABI’ retrieval process are: (1) 30 Principal Components (PC) scores are used as linear regression predictors for the PHS and MW all-sky Dual-Regression (DR) retrievals [1, 2] and GS ABI IR infrared radiances are used as regression predictors for the linear regression ABI retrievals; (2) all spatial samples, at the full spectral resolution of the PHS and ABI channels, are used to optimize the horizontal and vertical resolution of the PHSnABI fusion retrieval product [2], the fusion being performed using the very fast k-dimensional search-tree method [4]; (3) MW soundings [5] are fused with the IR soundings to fill-in the IR-profile gaps below clouds; (4) the IR and MW profiles are de-aliased to provide a vertical resolution comparable to the forecast model vertical resolution [6], the vertical alias removal being performed by computing the radiance spectrum from the 2-hr forecast Rapid Refresh (RAP) model profile valid at the time of the satellite observation. The ultra-fast PCRTM spectrum-based radiative transfer model [7] is used to define the vertical alias as the difference between the calculated RAP model radiance retrieval and the 2-hr forecast profile used to calculate the radiance model radiance spectra. Finally, (5) continuous NWP model assimilation of the satellite thermodynamic profile data is used to diagnose, through the numerical integration of the primitive equations of motion, 3-D horizontal and vertical wind velocities that correspond to the spatial and time variations of the satellite observations [8, 9, 10]. The data are used by a joint University of Wisconsin, NASA/LaRC, and Hampton University research team to produce high-resolution (i.e., 2-km spatial resolution and 30-minute temporal resolution) temperature and moisture profiles in near real-time for nowcasting and short-term numerical weather forecasts to provide warnings of localized intense storms as well as to conduct weather and climate research using high-resolution satellite sounding and operational weather data.

'PHSnMWnABI' moisture soundings are assimilated into a 4-km grid spacing Weather Research and Forecasting (WRF) model to produce hourly predictions of precipitation, severe weather (high wind, extreme rainfall, hail, and tornadoes) and other forecast model output variables [7, 9]. The initial condition for the forecast cycle is generated using a 2-hour period hourly interval assimilation of the 2-km resolution satellite soundings initialized using the operational RAP model analysis of conventional weather data. Three years of daily operation of the 'PHSnABI' and 'PHSnMWnABI' data production and assimilation system have shown significant, and consistent, improvements in the numerical prediction of CONUS region hazardous weather, particularly flood producing rainfall and tornados [9, 10].

3. Sounding and Forecast Characteristics

DeAliasing: Figure 1 shows the importance of enhancing regression retrieval vertical resolution to that of the model into which they are being assimilated. The vertical de-aliasing improves the agreement between the satellite derived profile and the radiosonde for both temperature and dewpoint temperature, reducing the dewpoint temperature differences by as much as a factor of 2. Most important is that the DRDA retrieval generally agrees better with the radiosonde than does the RAP model 2-hour profile used for the vertical alias removal process.

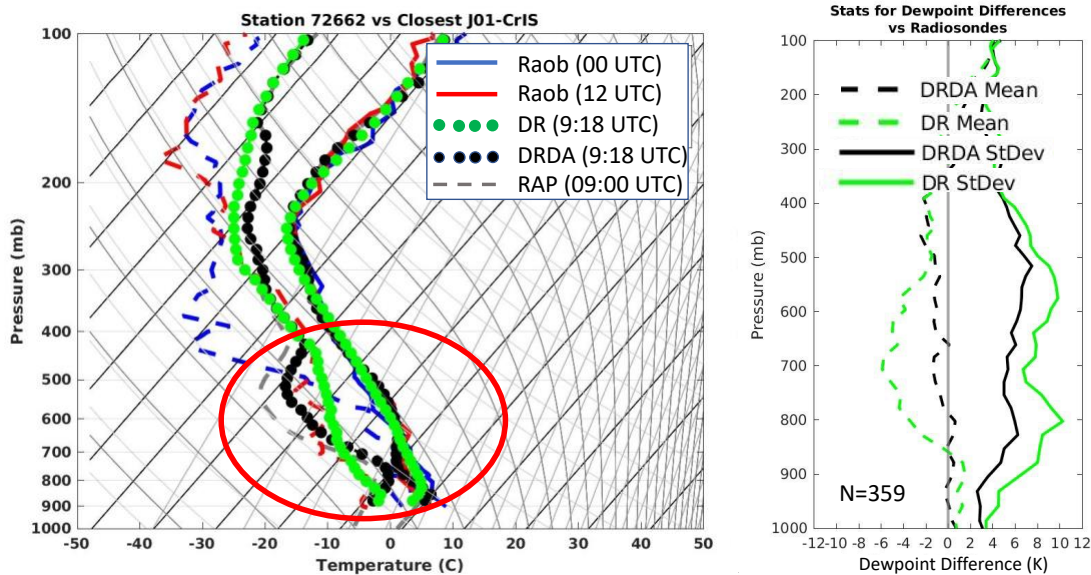


Figure 1. Comparison between Dual-Regression (DR) retrieval and De-aliased DR retrieval (DRDA), together with the model background profile (RAP 2-hour forecast) used for the alias removal, with a nearby radiosonde observation on November 11, 2021.

4. Fusion of Polar IR and Microwave Profiles with Geostationary Satellite IR Imager Soundings

The purpose of fusing PHS, and MW soundings with GS soundings is to produce fusion soundings (FS) which possess the vertical resolution and cloud penetrating information provided by polar satellites with the high temporal and spatial resolution information provided by geostationary satellites. The all-sky FS product provided on the UW website is produced for assimilation into numerical models. The soundings possess an all-sky spatial resolution of 2-km and an hourly temporal resolution. The procedure for performing the fusion of these data is summarized below.

(1) The first step of the fusion process is to spatially average the high horizontal resolution ABI soundings to the footprint areas observed of the CrIS and IASI polar hyperspectral sounding instruments. This provides a paired common area low-resolution ABI (i.e., LoresABI) and coincident polar hyperspectral

sounding training data set to be used to predict polar hyperspectral and microwave soundings at the locations and times of the full resolution ABI data. location and time to produce the hourly interval FS data. (2) Using a K-D search tree**, the 'N'=10 LoresABI soundings in the training data set, which provide the best agreement with each of the full resolution ABI soundings (i.e., HiresABI soundings), are selected. The parameters used for the best agreement selection order are the ABI regression retrieved RH sounding and associated RAP model temperature sounding as well as the latitude and longitude of the LoresABI sounding data. The value of 'N' is restricted by maximum allowable time and location differences (55-km, or 0.5 degrees, and 9-hours) between the PHS observations in the training data sample and the full resolution (Hires) ABI observation. (3) The weighted average of the PHS soundings in the training data set associated with the ordered by best agreement between the Hires ABI soundings and the LoresABI soundings in the training data set, as determined in (2), are then calculated for each ABI observation location and time. This weighted average PHS/LoresABI and MW /LoresABI sounding differences are then added to each ABI relative humidity and temperature sounding estimates to predict a Hires PHS and Microwave sounding at each ABI radiance measurement location and time.

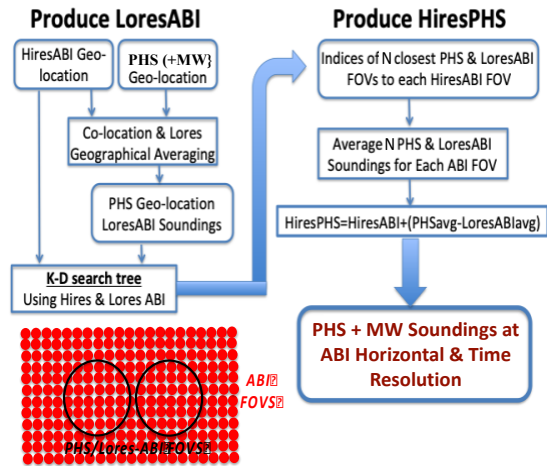


Figure 2. Schematic of the fusion process used to combine polar satellite soundings observations with geostationary satellite observations

5. Fusion Profile Examples

Figure 3 shows a comparison of two FS profiles showing vertical regions where the dewpoint temperature agrees better with the radiosonde than does the RAP 2-hr forecast, as is used in the polar sounding retrieval and geostationary satellite fusion process.

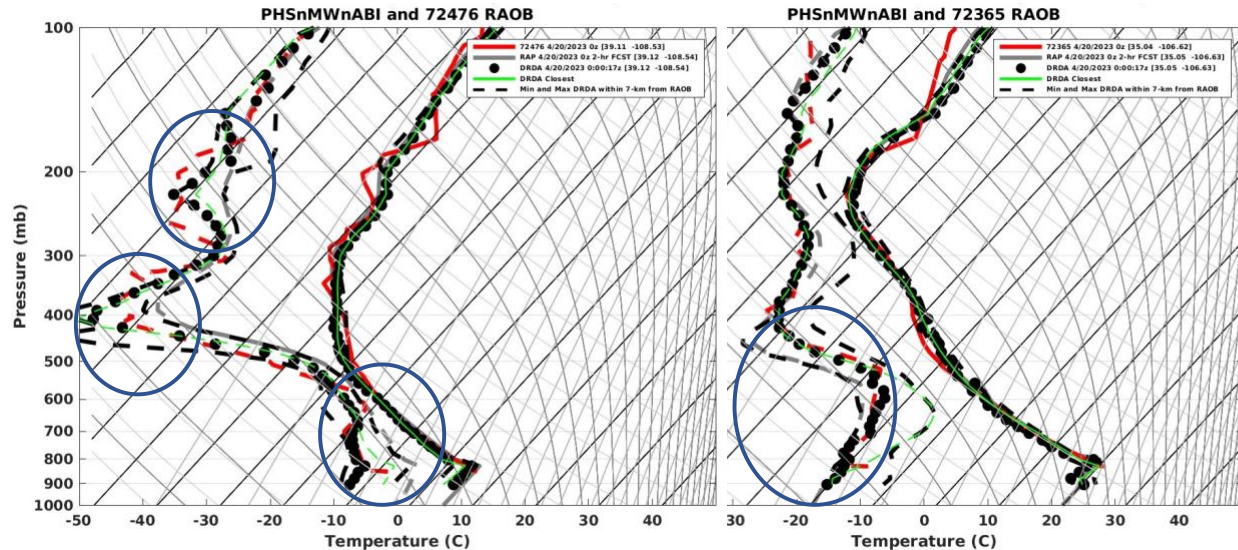


Figure 3. Comparison between FS profiles and radiosonde observations for 2 stations on April 20, 2023 (00 UTC), showing vertical regions where the dewpoint temperature agrees better with the radiosonde than does the RAP 2-hr forecast, as is used in the polar sounding retrieval and geostationary satellite fusion process.

6. NWP Model Assimilation

The model forecast model and data assimilation system is a 4-km resolution (grid-point spacing) of the NOAA RAP (Rapid Refresh) /HRRR (High Resolution Rapid Refresh) versions of the Weather Research and Forecast (WRF) model as described in [11]. The data assimilation system used is the Grid-point Statistical Interpolation System (GSI) analysis system [12]. As described by [14], a customized version of the WRF model [15, 16], and the GSI [17, 18], and [19] are being used. For the assimilation of the ‘PHSnMWnABI’ satellite soundings, no modifications are made to GSI source code. The UW-SSEC is running the version hosted by NOAA-EMC (<https://github.com/NOAA-EMC/GSI>). The control variable in minimization for water vapor is switched from mixing ratio to relative humidity. Only the satellite 2-km resolution retrieved relative humidity profile are being assimilated in the system (i.e., the satellite temperature profile and other retrieved cloud and surface parameters are not yet being assimilated). The observation relative humidity error covariance is updated on daily basis from the standard deviation between PHSnMWnABI profile retrievals and radiosonde observations (θ). The standard deviation (θ) of the differences between the retrievals and the radiosonde observations is also used for quality control. The embedded quality control in GSI is turned off. Instead, all water vapor profiles with ‘OmB’ (observation minus background) standard deviation smaller than 2θ are assimilated. Only one outer loop with 75 inner loops is used for cost function minimization.

A diagram showing the workflow of the PHSnMWnABI data assimilation and forecast system is shown in figure 4. As can be seen, an analysis cycle of the satellite profile data is initiated every hour using the RAP 13-km analysis as the background for the assimilation of the first of four sets of the satellite humidity profile retrievals using the GSI. The

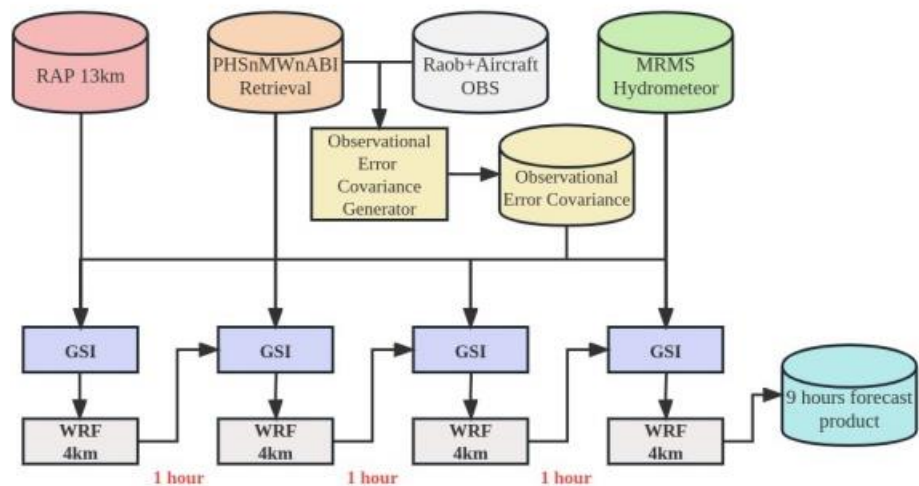


Figure 4. The workflow for the 4-km WRF model PHSnMnABI data assimilation system.

The analysis resulting from the first hour assimilation is used to produce a 4-km resolution WRF forecast which is then used to provide the background for a 2nd hour assimilation of satellite humidity profile retrievals from which a second 1-hr WRF forecast is made. This process continues for two more hours so that a total of all satellite soundings observed during a 4-hour period are assimilated to produce the final analysis, which is used to initiate a nine-hour forecast cycle. The continuous assimilation of the water vapor profile data enables the wind velocity profiles, associated with the satellite derived atmospheric moisture profiles, to be derived for the initialization of the model forecast cycle [9,13]. The quasi-continuous assimilation of these satellite moisture profile data enables the model dynamics (i.e., winds) to adjust to the thermodynamic constraints of the moisture assimilation through the time integration of the model’s equations of motion. Multi-Radar Multi-Sensor (MRMS) observations are also used to initialize a 9-hour WRF model forecast cycle. It is noted that the RAP background used to initialize the 3-hour long continuous data assimilation cycle has already assimilated all the conventional and satellite

sounding radiance information, as well as radar and other operational satellite remote sensing observations that are assimilated into the NOAA’s operational RAP/HRRR forecast systems. Therefore, the 4-km WRF model forecasts benefit from all the operational meteorological data, as well as from the high spatial/temporal resolution polar/geostationary satellite fusion soundings assimilated to improve numerical forecasts of convective severe weather.

7. Website Displays

Currently the PHSnMWnABI sounding retrieval product plots are available every hour at <https://www.ssec.wisc.edu/hufusion/data#plot-viewer/> for a domain extending from 20 N to 50 N latitude and 70W to 160 W longitude. The website for accessing displays of the PHSnMWnABI sounding data can be viewed at: <https://www.ssec.wisc.edu/hufusion/>. Plots are shown for the temperature and relative humidity at three pressure levels (i.e., 850-hPa, 700-hPa, and 500-hPa) and for relative humidity at three upper tropospheric levels (400-hPa, 300-hPa, and 200-hPa). Also shown are the Lifted Index stability parameter, cloud top pressure, and surface skin temperature.

Plot Viewer: These displays can be seen at: <https://www.ssec.wisc.edu/hufusion/data#plot-viewer/>. As an example, figure 5 shows the panels of the relative humidity displayed on the plots for May 11, 2023,

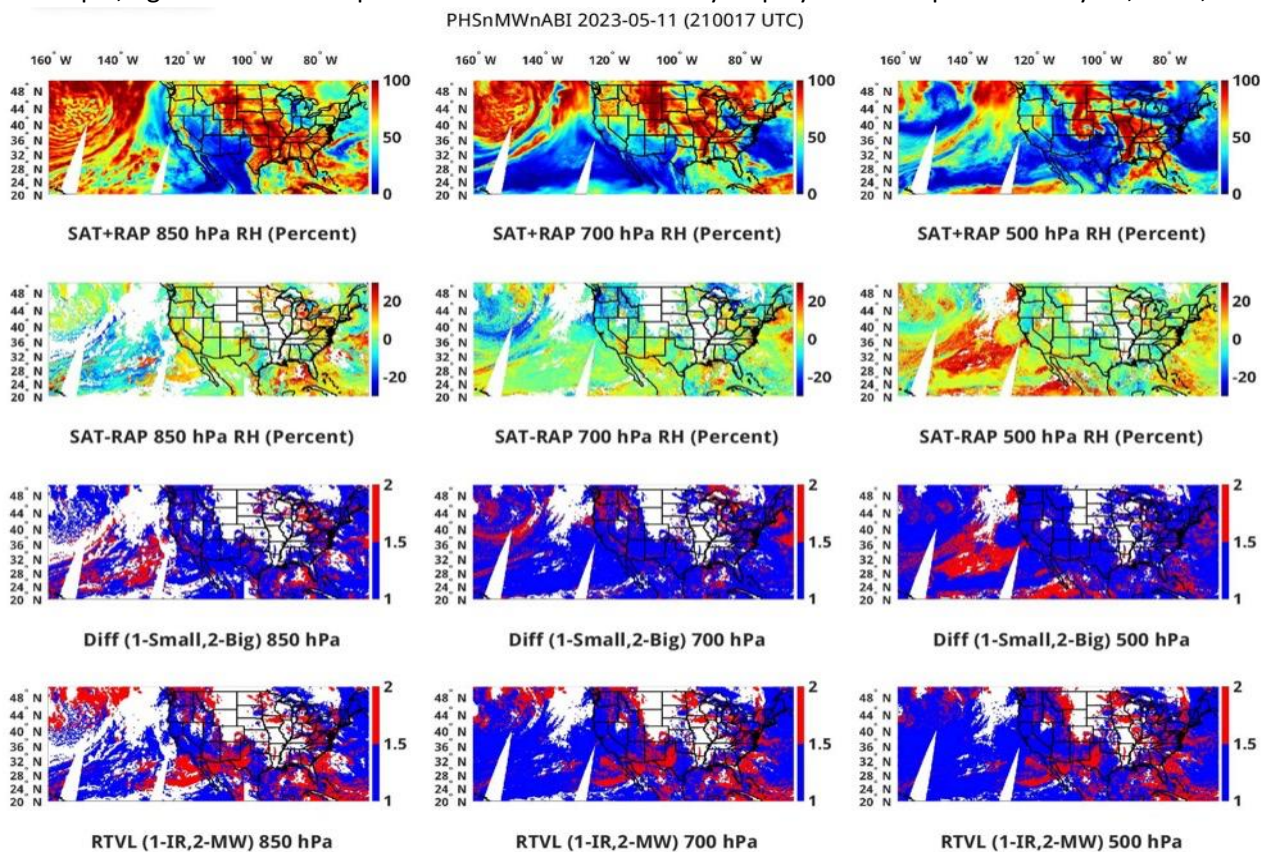


Figure 5. PHSnMWnABI relative humidity for 850-hPa, 700-hPa, and 500-hPa. The top row of panels show the combined satellite and RAP profile data, the second row the difference between the satellite and RAP data, the third row shows whether the difference is small (blue) or large (red) and the fourth row of panels show whether the retrieval value was defined using the infrared hyperspectral or the microwave radiance data from the polar satellites.

at 21 UTC. As shown, there are four different panels for each pressure level shown. The SAT+RAP color analyses is obtained by plotting the satellite retrieval data where it exists and the RAP 2-hr forecast data where the retrievals are missing. The satellite soundings may be missing due to the lack of polar satellite coverage or where the infrared or microwave retrieval data is missing due to clouds, precipitation, or high terrain.

Radiosonde Comparisons: PHSnMWnABI profile retrievals are compared to NUCAPS, Radiosondes, and the RAP 2-hr forecast profiles used for the PHSnMWnABI retrievals (<https://www.ssec.wisc.edu/hufusion/data#radiosonde/>). For the NUCAPS comparisons, the 2-km spatial resolution 'PHSnMWnABI' (DRDA) retrievals are averaged over the 50-km Field-Of-Regard (FOR) of the NUCAPS soundings. For the comparison of the full 2-km resolution PHSnMWnABI retrievals with radiosonde comparisons, the retrieved profile within 7-km of the radiosonde station location, whose 150-hPa layer average relative humidity best matches that of the radiosonde, is selected for the comparisons to account for the local air mass variations of humidity (e.g., produced by clouds) and radiosonde balloon drift away from the launch location. Also shown are the retrieved sounding minimum and maximum radiosonde difference values of all the sounding retrieval values within the 7-km radius centered on the radiosonde launch site. Figure 6 shows an example comparison with the Norman OK radiosonde observed on May 6, 2023.

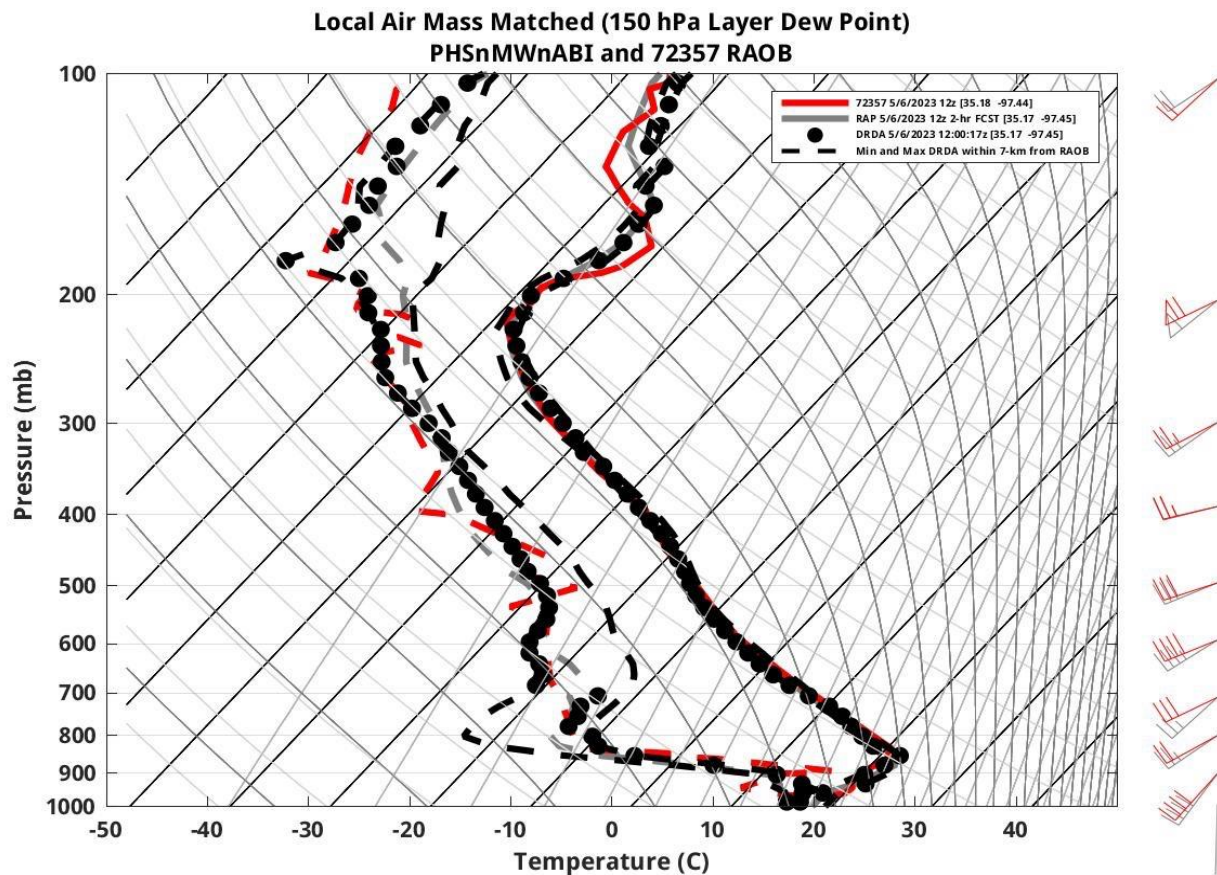


Figure 6. Comparison between the fusion PMnA (i.e., DRDA) retrieval at 12 UTC on May 6, 2023, at Norman Oklahoma. The black circles (DRDA) show the the PMnA sounding within a 7-km radius of the radiosonde location whose 150-km vertical average humidity best matches the vertical average of the radiosonde observation. The gray lines show the 2-hr RAP forecast sounding used for the retrieval. The dashed black lines show the range of all retrievals within a 7-km radius of the radiosonde station.

8. Forecast Plots

The forecast plots can be found at: <https://www.ssec.wisc.edu/hufusion/forecast-plots/> The use of the display tool is self-explanatory. Basically, the URL takes you to a calendar for the user to select the day of the month for which the forecast parameters are desired. After clicking on the day, another menu will be presented where the user can choose, using the pull-down menu, the Forecast Initialization Time for which the 0-to 9-hour forecasts that were made using that initialization time. The 0-hour forecast is the final analysis used to initialize the forecast cycle. After clicking on the initialization time desired, nine panels of forecast parameters, each panel containing 14 different forecast parameters: BWD01, BWD03, and BWD06 (Bulk Wind Shear for the Surface to 1000 m, Surface to 3000m, and surface to 6000m layers, respectively), EHI (Energy Helicity Index), Hel01, HEL03

Forecast Plots

2023

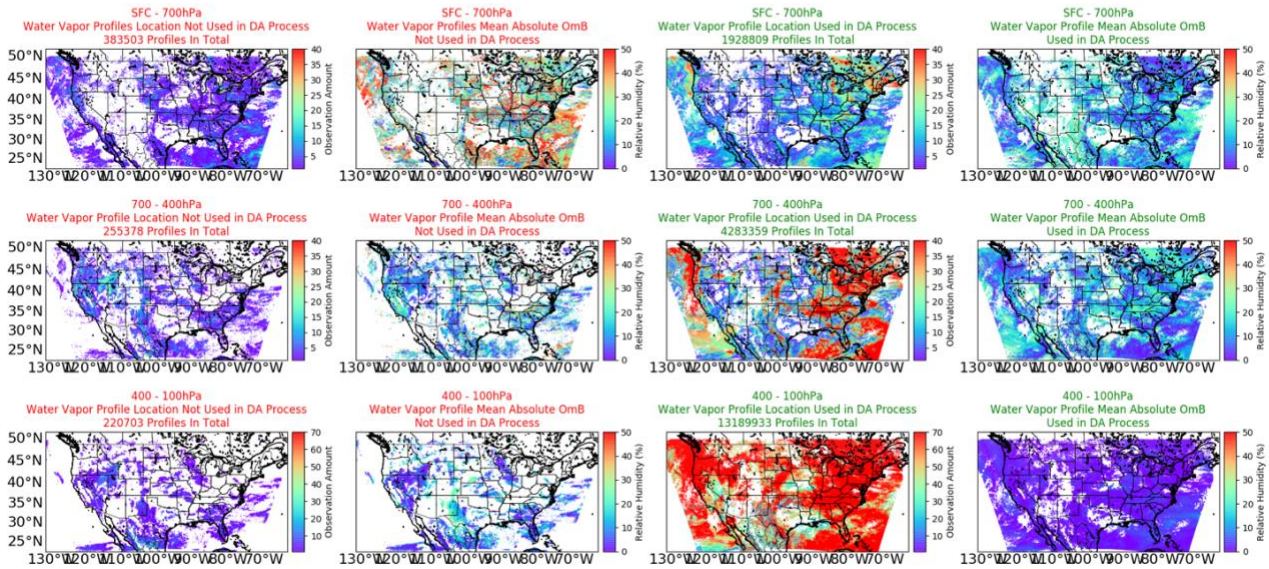
May 2023						
Sun	Mon	Tue	Wed	Thu	Fri	Sat
	1	2	3	4	05	06
07	08	09	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30	31			

(Storm Relative Helicity for the Surface to 1000m, and Surface to 3000m layers, respectively), LCL

The screenshot displays two side-by-side forecast panels. The left panel is titled 'Model: oper_hrrr3km_central' and the right panel is titled 'Model: ssec_wrf4km_eastern'. Both panels show a grid of forecast parameters for the date 2023-05-10. The parameters listed on the y-axis are BWD01, BWD03, BWD06, EHI, HEL01, HEL03, LCL, LFC, MUCAPE, SBCAPE, SCP, SHIP, STP, and mesoanalysis. The x-axis represents initialization times from 01Z to 09Z. Each cell in the grid contains a small circular icon, likely representing a forecast value or status for that parameter and time.

(Lifted Condensation Level), LFC (Level of Free Convection), MUCAPE (Most Unstable CAPE), SBCAPE (Surface-based CAPE), SCP (Supercell Composite Parameter), SHIP (Significant Hail Parameter), and STP (Significant Tornado Parameter). These parameters are defined from the forecast atmospheric state parameters produced by two models, (1) NOAA’s Operational High Resolution Rapid Refresh (HRRR) model and (2) the experimental satellite sounding data assimilation WRF model (section 6). These parameters defined from the model forecasts are shown for the NWS Central, Eastern, Southern, and Western CONUS regions. The last option provided by the forecast display tool is the “Model: wrf4km_diagnostic” which displays maps for three atmospheric layers (surface to 700-hPa, 700-hPa to 400-hPa, and 400 to 100-hPa) the relative amount of the high-resolution satellite ‘PHSnMWnABI’ water vapor profile data “Not Used” and “Used” because of the GSI data assimilation (DA) process. Also shown, for each atmospheric layer, are the Observation minus Background (OmB) differences of the Relative Humidity values rejected and accepted by the GSI system. The background used for the ‘OmB ‘calculation is the final 1-hour WRF forecast of the 3-hour long, hourly interval, satellite relative humidity profile data assimilation cycle.

Model: wrf4km_diagnosic		
VARS	2023-05-10	
	00Z	
	ProfileInuse	▲
		●
Diag	0	



9. Fusion Sounding Map

The website also contains an application for the user to see the difference between the PHSnMWnABI (called PMnABI) profile retrieval and the RAP 2-hr forecast profile that was used as the background for the retrieval. The fusion sounding map, accessed from: <http://cas.hamptonu.edu/~adinorscia/InteractiveMap/FusionMap.html>. An example displaying the Fusion Map tool is shown here. There are 3 pull down menus: one to select the day (today or yesterday), another to select the hour (00 UTC to 2300 UTC), and the third to select the background image (Lifted Index, Cloud Top Pressure, surface skin temperature, and the Relative Humidity difference (Sat-RAP) for either the 850-hPa, 700-hPa, or 500-hPa levels). Clear regions can be seen by selecting the 'Cloud Pressure' option, where the clear pixels are shown by the white missing data pixels and the clouded regions can be seen by selecting the 'Surface Skin Temperature' map where the cloudy pixels are shown by the white missing data pixels. It must be remembered to always click on the 'Load Map and Sounding' bar when any selection is changed. After loading the background map, the user can use the cursor to select a geographical position to display a Skew-T plots of the average of all 'PHSnMWnABI' temperature and dewpoint profiles, and associated 2-hour RAP forecast profiles, within a 40-km radius of the geographical point selected. The Lifted Index and MUCAPE values are also shown on the Skew-T plots.

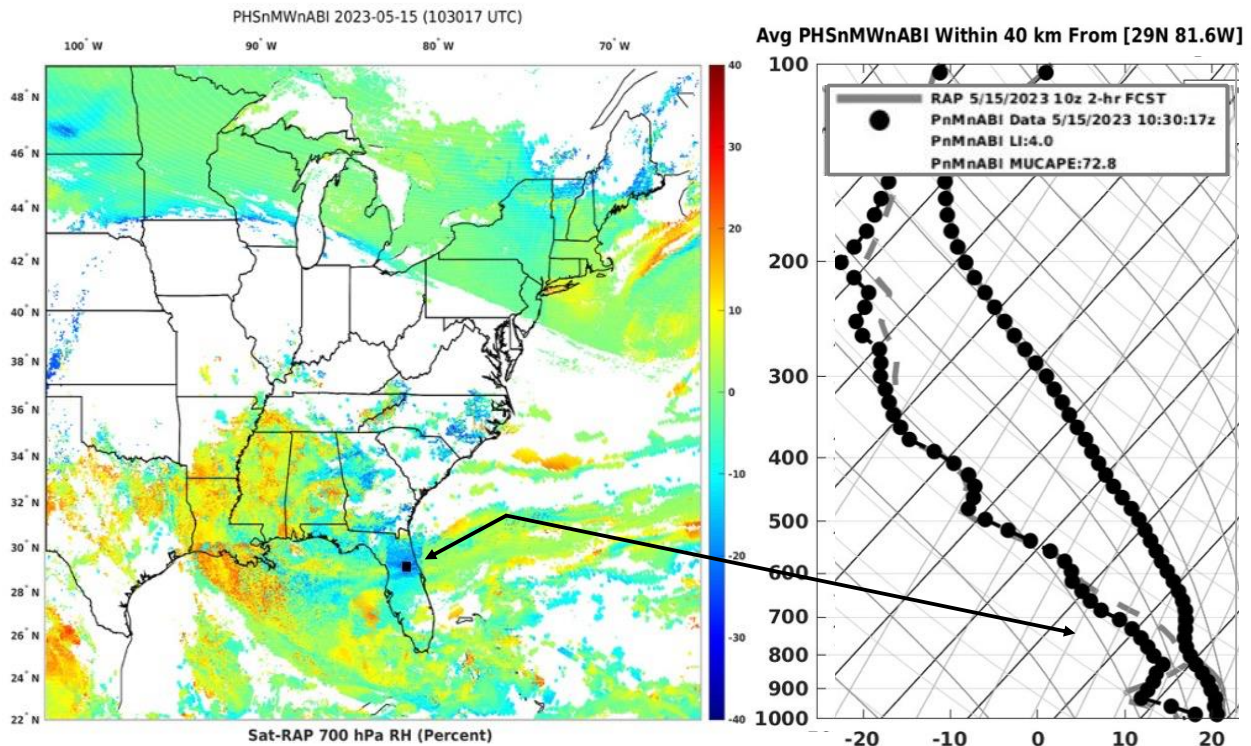
Severe Weather/Tornado PHSnABI Sounding Map

Load a map and click anywhere for that location's 25-km average sounding:

Choose for Today's or Yesterday's Data: Today Choose Hour: 10z

Choose Atmospheric Parameter for Map: 700 hPa RH (%)

Load Map and Soundings



10. References

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